ST. XAVIER'S SENIOR SECONDARY SCHOOL, DELHI – 110 054

Class 11 Time: 3 hrs.

19-9-2023 MID TERM EXAMINATION - **MATHEMATICS** Max. Marks : 80

General Instructions:

i) This question paper consists of 38 questions divided into four sections A, B, C, D and E. All questions are compulsory.

ii) Section A consists of 18 M.C.Q and 02 assertion reason based questions carrying 1 mark each. Section B consists of 5 very short answer questions carrying 2 marks each. Section C consists of 6 short answer question carrying 3 marks each. Section D consists of 4 long answer questions carrying 5 marks each. Section E has 3 source based/case based questions carrying 4 marks each.

iii) There is no overall choice. However, an internal choice has been given in some questions.

SECTION – A

1. Set A = {0}, set B = {x: x > 15 and x< 5}, set C = {x: x-5=0}, set D = {x: x^2=25} set E={x: x is integral positive root of the equation x^2-2x-15=0}. Which of the following is true:
   a) A=D b) B=E c) C=D d) C=E

2. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} is a universal set and P = {1, 2, 5}, Q = {6, 7}, then P′ ∩ Q is:
   a) P b) Q c) Q′ d) P′

3. The number of subsets that can be formed for set A = ∅ is:
   a) 0 b) 1 c) 2 d) 4

4. The range of $f(x) = x^2 + 2$ is
   a) [-2, ∞) b) (-∞, 2] c) [2,∞) d) [1,∞)

5. The domain of the function $f(x) = \sqrt{x-1} + \sqrt{3-x}$ is:
   a) [1, ∞) b) (-∞, 3) c) (1, 3) d) [1, 3]

6. If $f(x) = \frac{x-3}{x-1}, (x \neq 1)$, then $f[f(x)]$ is equal to:
   a) x b) x^2 c) $\frac{1}{x}$ d) 0

7. The value of $\frac{\sin A + sin3A}{cosA + cos3A}$ is equal to:
   a) $\cot 2A$ b) $\tan 2A$ c) $\cot 3A$ d) $\tan 3A$

8. The value of $4\cos\frac{\pi}{6}\sec\frac{\pi}{6} - 4\cos\frac{5\pi}{6}\cot\frac{\pi}{3}$ is equal to:
   a) 1 b) 2 c) 6 d) -2

9. If $\sin A = \frac{3}{5}$, $\cos B = \frac{-12}{13}$, where A and B both lie in second quadrant, then the value of $\sin(A + B)$ is:
   a) $\frac{-28}{65}$ b) $\frac{28}{65}$ c) $\frac{-56}{65}$ d) $\frac{56}{65}$

10. If $4x + 3 < 6x + 7$ then the value of $x; \forall\, x \in Z$ is:
    a) [-2, ∞] b) {-2, -1, 0, 1…..}
    c) $\{-1, 0, 1, 2 \ldots\ldots\}$ d) $(-2, \infty)$

11. The solution set of $\frac{-2}{x+4} > 0$ is:
    a) [-∞, 4] b) (-4, ∞) c) $(-4, 4)$ d) $(-\alpha, -4)$

12. The marks scored by Rohit in two test were 65 and 70. If Rohit scored $'x'$ marks in the third test to achieve an average of at least 65 marks, then which of the following is correct:
a) $x > 65$ b) $x < 60$ c) x = 55 d) $x \geq 60$

13. The value of $\frac{1}{i^{99}}$ is:
a) i b) $-i$ c) 1 d) -1

14. The value of $\sqrt{-25} \times \sqrt{-9}$ is
a) 15 b) $15i$ c) -15 d) $-15i$

15. The modulus of $\frac{(1+i)(1+\sqrt{3}i)}{1-i}$ is:
a) 3 b) 2 c) $2\sqrt{2}$ d) 4

16. The geometric mean of a^3b and ab^3 is:
a) $\frac{ab(a^2+b^2)}{2}$ b) a^3b^2 c) a^2b^2 d) ab

17. Which term of G.P $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}} \ldots\ldots\ldots$ is $\frac{1}{512\sqrt{2}}$:
a) 2^{th} b) 11^{th} c) 10^{th} d) 8^{th}

18. In a G.P. the 3^{rd} term is 24 and 6^{th} term is 192. Then its 10^{th} term is:
a) 3072 b) 3074 c) 2072 d) 2074

In questions 19 and 20, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer from the given choices.

19. Assertion(A): The collection of all natural numbers less than 100 is a set.
Reason(R): A set is a well-defined collection of distinct objects.
a) Both A and R are true and R is correct explanation of A.
b) Both A and R are true but R is not a correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. Assertion (A): For $x = \pm 1$, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P
Reason (R): Three numbers a, b, c are in G.P. if $b^2 = ac$.
a) Both A and R are true and R is correct explanation of A.
b) Both A and R are true but R is not a correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

SECTION – B

21. Let U={x:x ∈N, x ≤ 9}; A={x:x is even number, 0≤ x<10}; B={2,3,5,7}. Write the set (AUB)'.

22. Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a function from Z to Z defined by $f(x) = (ax + b)$, for some integers a and b. Find a, b.

23. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use $\pi = 3.14$)

24. Find two numbers whose A.M is 34 and G.M is 16.
(OR)
Find the sum to n terms of the sequence 7, 77, 777, 7777, …… to n terms.

25. Solve the inequation: $-5 \leq \frac{2-3x}{4} \leq 9$ and write the solution set.

SECTION – C

26. Prove that $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2=4\cos^2\left(\frac{\alpha-\beta}{2}\right)$.

27. If $\tan A=\frac{m}{m-1}$ and $\tan B=\frac{1}{2m-1}$. Prove that $A-B=\frac{\pi}{4}$.

(OR)

If $\sin x=-\frac{1}{2}, x$ lies in IV quadrant, find $\sin\frac{x}{2},\cos\frac{x}{2}\ and\ \tan\frac{x}{2}$.

28. If $(x+iy)^{\frac{1}{3}}=(a+ib)$, prove that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$.

(OR)

Let $Z_1=2-i$ and $Z_2=-2+i$.
Find (i) $\mathrm{Re}\left(\frac{z_1z_2}{\overline{z_1}}\right)$ (ii) $\mathrm{Im}\left(\frac{1}{z_1z_2}\right)$

29. Let $f(x)$ and $g(x)$ be real valued functions defined by $f(x)=\sqrt{x+2}$ and $g(x)=\sqrt{4-x^2}$. Then, find a function $\emptyset(x)=(fg)(x)$. Write the domain of $\emptyset(x)$.

(OR)

Let A={1,2,3,5} and B={4,6,9}. Define a relation R from A to B by
$R=\{(x,y)$: the difference between x and y is odd, $x\in A, y\in B\}$. Write the domain and range of R.

30. Find the real value of x and y for which the complex numbers $(x+iy)(2-3i)$ and $4-i$ are conjugate of each other.

31. The length of a rectangle is five times its breadth. What is the minimum length of the rectangle so that the perimeter of rectangle is atleast 120cm.

SECTION – D

32. Let S be the sum, P be the product and R be the sum of reciprocals of n terms of a G.P. prove that $P^2R^n=S^n$.

(OR)

If pth, qth and rth terms of a G.P are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}c^{p-q}=1$.

33. Prove that $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$.

(OR)

Prove that $\frac{\cos 8A\cos 5A-\cos 12A\cos 9A}{\sin 8A\cos 5A+\cos 12A\sin 9A}=\tan 4A$.

34. If α and β are different complex numbers with $|\beta|=1$, find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$.

35. Prove that: $\frac{\sec 8x-1}{\sec 4x-1}=\frac{\tan 8x}{\tan 2x}$.

SECTION – E

36. A mathematics teacher Mamta Sharma of class XI write the following sets on a black-board U = $\{x: x=n, n\in N, n\le 15\}$, A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8} and C = {2, 3, 5, 7, 11}
Based on above information answer the following :

i) Find (A∪B)∩(A∩B)
ii) Find (A-B)∪(B-A)
iii) Verify the relationship (A∪B)′ = A′∩B′

(OR)

Draw the venn diagram from above information.

37. In a sequence every even term is 'a' times the term before it and every odd term is 'c' times the term before it. The first term of the sequence is unity.
    i) Write first five terms of the sequence.
    ii) Write 7^{th} term of the sequence.
    iii) Find the sum of 2n terms of the sequence.

38. A company manufactures cassettes. Its cost price and selling price functions for the week are given by C(x)= 300 + $\frac{3}{2}x$ and S(x)= $2x$ respectively, where x is the number of cassettes produced and sold per week.
    i) Write the profit function P (x) for above transaction.
    ii) How many cassettes must be sold in a week to realize a profit?

-x-x-x-x-x-x-x-x-